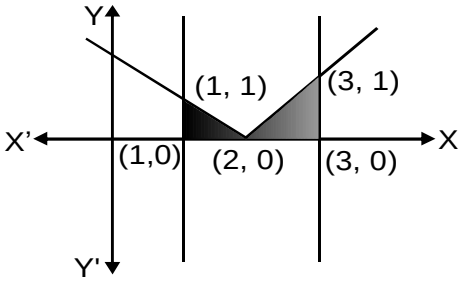
AREA UNDER THE CURVE

EXERCISE – I

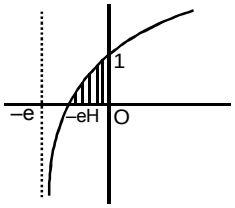
HINTS & SOLUTIONS

Sol.1 **C**



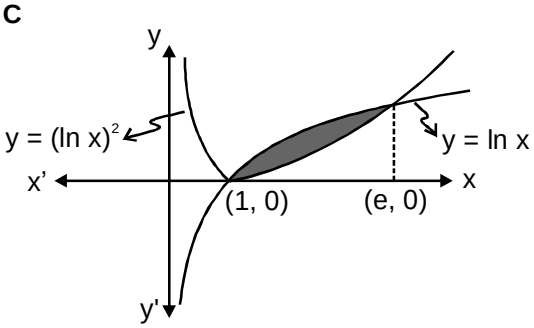
Area $A = \frac{1}{2} \times 1 \times 1 + \frac{1}{2} \times 1 \times 1 = 1$

Sol.2 **D**



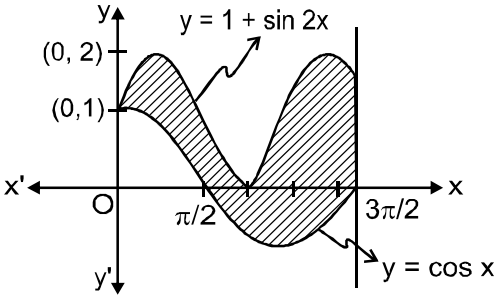
$A = \int_0^1 (e^y - e)\, dy = 1$

Sol.3 **C**



$A = \int_1^e (\ell n^2 x - \ell n\, x)\, dx = 3 - e$

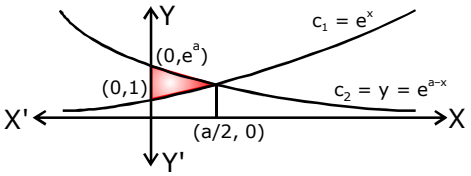
Sol.4 **C**



$A = \int_0^{3\pi/2} (1 + \sin 2x - \cos x)\, dx$

$A = \int_0^{3\pi/2} (1 + \sin 2x - \cos x)\, dx = 2 + \frac{3\pi}{2}$

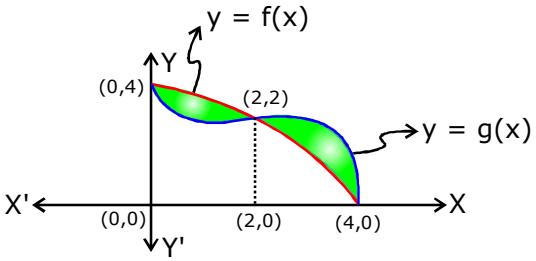
Sol.5 **D**



$S = \int_0^{a/2} (e^a e^{-x} - e^x)\, dx$

& $\lim_{a \to 0} \frac{S}{a^2} = \lim_{a \to 0} \frac{e^a - 2e^{a/2} + 1}{a^2} = \frac{1}{4}$

Sol.6 **C**

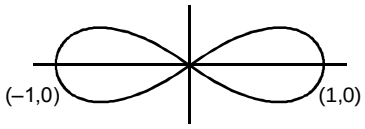


$\int_0^4 (f(x) - g(x))\, dx = \int_0^2 (f(x) - g(x))\, dx + \int_2^4 (g(x) - f(x))\, dx$

$\int_0^2 (f(x)\, g(x)\, dx = 15$

Sol.7 **B**

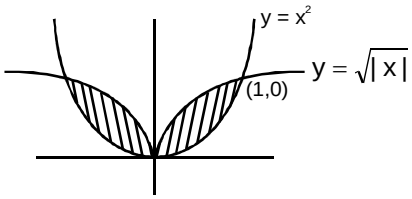
curve is symmetric about both the axes & cuts x-axis at (–1, 0) (0, 0) & (1, 0)



Area of loop $= 2 \int_0^1 x\sqrt{1 - x^2}\, dx$

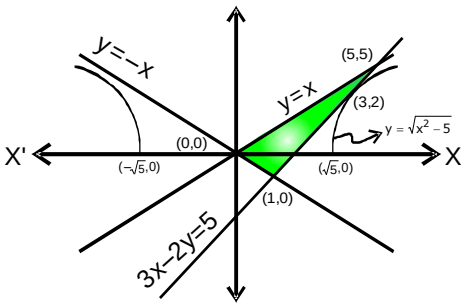
$= 2 . \frac{2}{3} = \frac{4}{3}$

Sol.8 B



$A = 2\int_0^1 \left(\sqrt{|x|} - x^2\right) dx = \frac{2}{3}$

Sol.9 A



Equation of tangent area of shaded region

$= \frac{1}{2}\left|5(-1) - 5(1)\right| = 5$

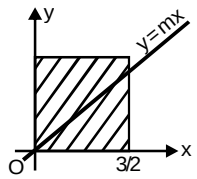
Sol.10 A

$\frac{d^2y}{dx^2} = 0$ at x = 2 so $A = \int_0^2 xe^{-x}\, dx = 1 - 3e^{-2}$

Sol.11 A

$A = \int_0^{3/2} y\, dx = \frac{39}{8}$

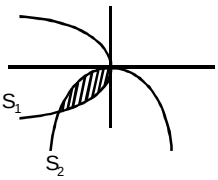
and $\left(\frac{39}{8}\right) \times \frac{1}{2} = \int_0^{3/2} mx\, dx$



$\Rightarrow$ m = 13/6

Sol.12 B

Area of shaded region = $\frac{1}{3}$



Sol.13 A

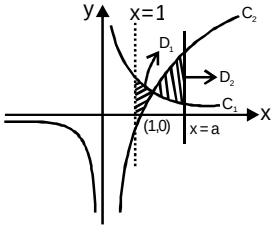
$\sin 2x - \sqrt{3}\ \sin x = 0 \Rightarrow \sin x \left(\cos x - \frac{\sqrt{3}}{2}\right) = 0$

x = 0 on $\pi/6$

so $A = \int_0^a (\sin 2x - \sqrt{3}\sin x)\, dx$

$\Rightarrow$ 4A + 8 cos a = 7.

Sol.14 B



Sol.15 C

$\int_1^b f(x)\, dx = (b-1)\sin(3b+4)$

differentiate w.r.t. 'b'

f(b)·1 = 3(b – 1) cos (3b + 4) + sin (3b + 4)

so f(x) = 3(x – 1) cos (3x + 4) + sin (3x + 4)

Sol.16 C

$A = \int_0^1 (3 - 2x - x^2)\, dx$

Sol.17 B

$A = \int_{1/2}^{e} (x(1 - \ell n\, x))\, dx = \frac{e^2 - 5e^{-2}}{4}$

Sol.18 B

from at point (1, 3)

A + B + C = 3 ....(i)

equation of tangent at (2, 0)

y = 4Ax + Bx + 2B + 2ℓ ....(ii)

comparing with 4x + y = 8 (given tangent)

get A, B, C & area.

Sol.19 D

$A = 5 - \int_0^1 (3x^3 + 2x)\, dx = \frac{13}{4}$

Sol.20 A

$\frac{dy}{dx} - y = \cos x - \sin x$

I.F. = e^{-x} so $y.e^{-x} = \int e^{-x}(\cos x - \sin x)\, dx$

$y.e^{-x} = e^{-x}\sin x + c$

ax $x \to \infty$, c = 0 so y = sin x

Area inclosed by y = sin x, y = cos x & y-axis t in Ist quadrant

$= \sqrt{2} - 1$